Université d'Ottawa
Département de
génie mécanique

University of Ottawa
Department of
mechanical engineering

Assignment/Devoir 8

MCG 3340/3370, Fall/Automne, 2016

MCG 3340, Not to be turned in **MCG 3740, Pas à remettre**

Question 1:

A piping system is used to deliver water from a lake to a storage tank, as shown. The piping system is made of commercial steel and is 20 m long in total. The pipes have an inner diameter of 4 cm. The system has an electric pump and three 90° elbows. The pressure difference provided by the pump is 66 kPa. The viscosity of the water is, $\mu_{H_2O} = 1.3 \times 10^{-3}$ Pa s. Treat the pipe entrace as "reentrant" (see page 11).

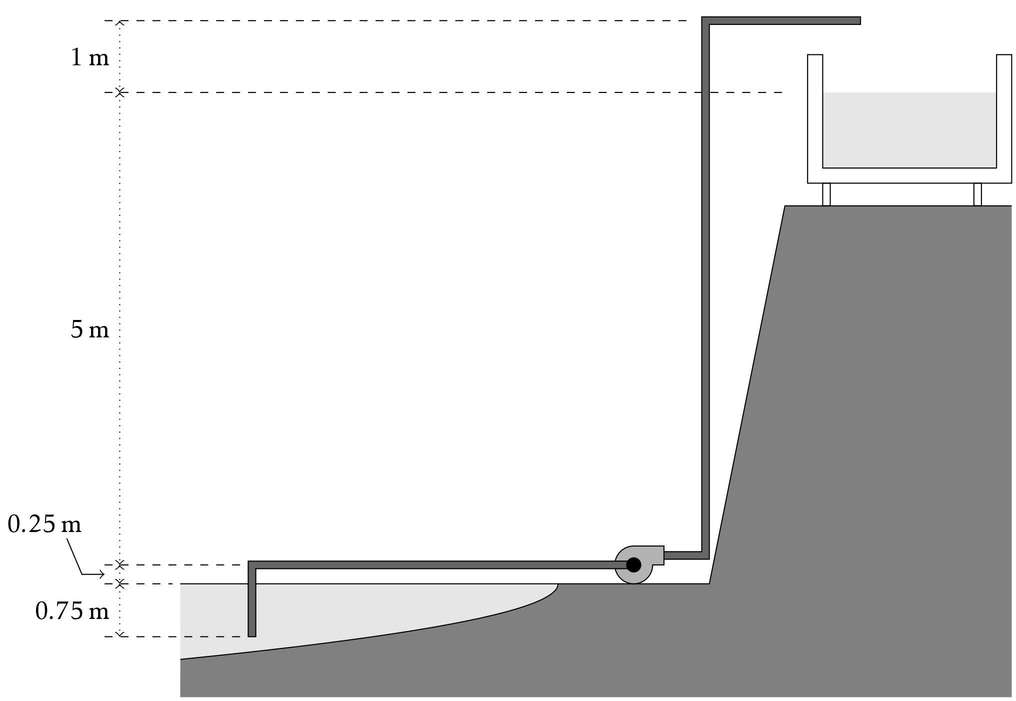
a) What is the expected volumetric flow rate?

b) If the pump has an efficiency of 75%, how much power does it consume?

Un système de tuyaux transporte de l'eau d'un lac à un récipient. Les tuyaux sont fabriqués d'acier comercial (commercial steel) et ont une longueur totale de 20 m. Le diamètre est de 4 cm. Le système contient une pompe électrique et trois coudes à 90 degrés (90° elbows). La difference de pression fournie par la pompe est de 66 kPa. La viscosité de l'eau est $\mu_{H_2O} = 1{,}3 \times 10^{-3}$ Pa s. Traitez l'entrée du tuyau comme être "reentrant" (voir page 11).

a) Quelle est le débit volumetrique attendu?

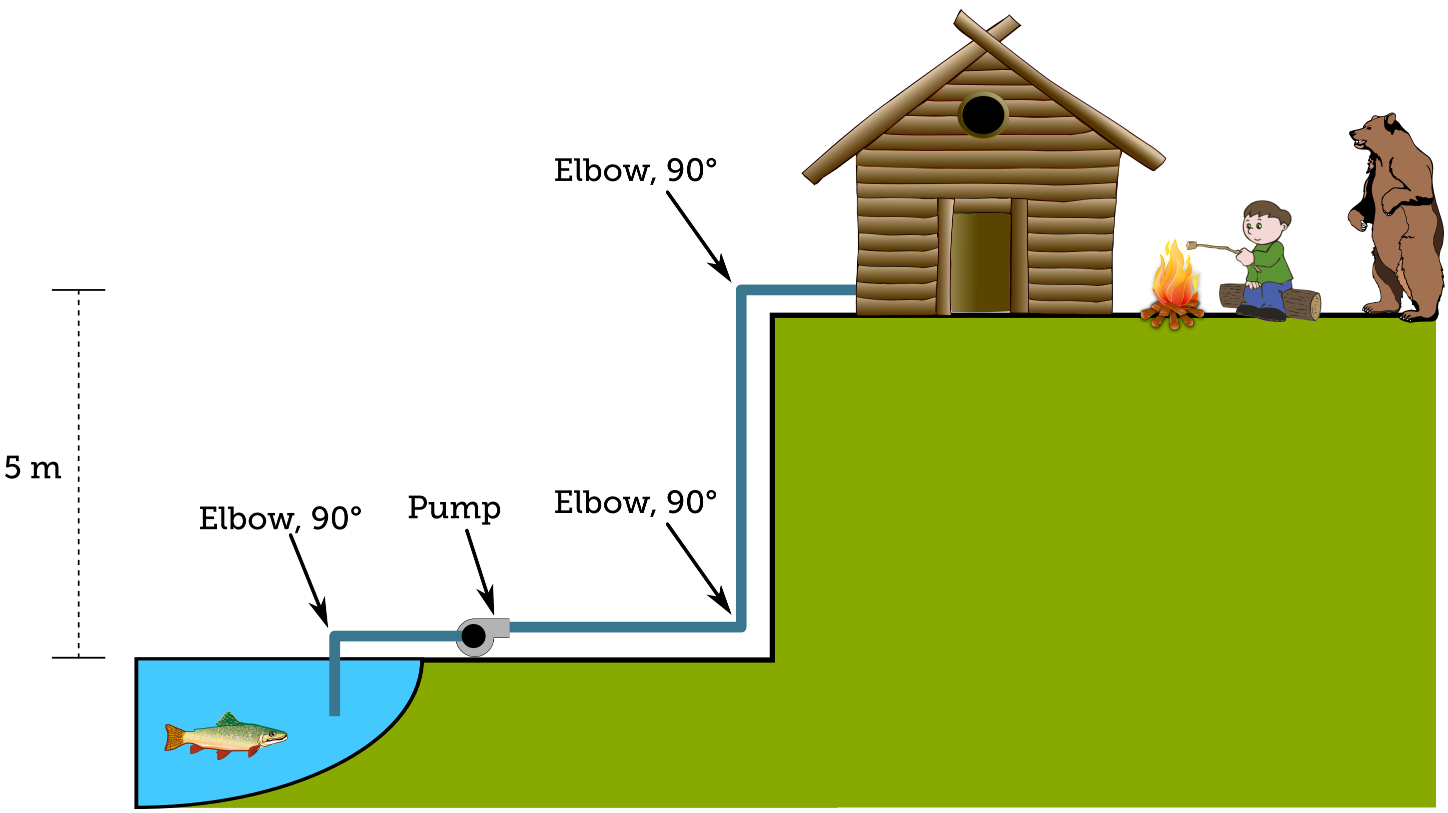
b) Si la pompe a une efficacité de 75%, quelle puissance est-ce qu'elle consume?



Question 2:

Un système de tuyaux transporte de l'eau d'un lac à une cabane. La cabane est 5 mètres plus haut que la surface du lac. Les tuyaux sont fabriqués de l'acier comercial (commercial steel) et ont une longeur totale de 30 mètres. Le diamètre est de 2,5 centimètres. Le système contient une pompe électrique et trois coudes à 90 degrés (90° elbows). Le conduit fournit de l'eau à la cabane à une pression relative de 300 kPa avec un débit volumique de 20 litres par minute. La viscosité de l'eau est $\mu_{H_2O} = 1,3 \times 10^{-3}$ Ns/m^2. La pompe a une éfficacité de 75%.
Quelle puissance électrique doit être fournie à la pompe?
Négligez la perte locale à l'entrée du tuyau.

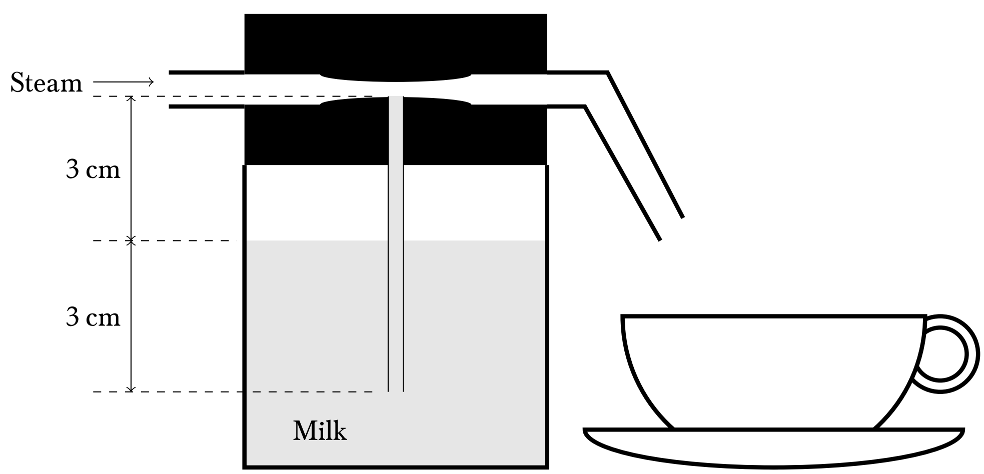
A piping system is used to deliver water from a lake to a cabin, which is situated 5 meters higher than the lake level. A 30-m long piping system made of commercial steel is used, with an internal diameter of 2.5 cm. The system has three 90° elbows. The delivery water pressure required at the cabin entrance is 300 kPa (gauge) and the required volume flow rate is 20 liters per minute. The water viscosity is $\mu_{H_2O} = 1.3 \times 10^{-3}$ Ns/m^2. The pump has an efficiency of 75%.
What electrical power does the pump require?
Neglect minor losses at the pipe entrance.



Question 3:

An automatic cappuccino machine uses a Venturi to draw milk, $\rho_{\text{milk}} = 1035\,\text{kg/m}^3$, vertically into a stream of hot steam through a straw with a diameter of 0.5 cm and a length of 6 cm. The viscosity of the milk is $\mu_{\text{milk}} = 3 \times 10^{-3}\,\text{Pa s}$. The steam has a density, $\rho_{\text{steam}} = 1.7\,\text{kg/m}^3$ and it enters the Venturi with a relative pressure of 200 Pa and velocity of 3 m/s. Assume the loss coefficient, K, at both the straw entrance and exit is 0.5. Neglect viscosity in the steam flow. What must be the ratio between the maximum and minimum diameters in the Venturi to achieve a milk flow of 12 ml/s?

Une machine à cappuccino automatique utilise un Venturi pour tirer du lait, $\rho_{\text{lait}} = 1035\,\text{kg/m}^3$, verticalement dans un flux de vapeur chaude à travers une paille avec un diamètre de 0,5 cm et une longueur de 6 cm. La viscosité du lait est de $\mu_{\text{milk}} = 3 \times 10^{-3}\,\text{Pa s}$. La densité de la vapeur est de $\rho_{\text{steam}} = 1{,}7\,\text{kg/m}^3$. La vapeur entre dans le Venturi avec une pression relative de 200 Pa et une vitesse de 3 m/s. Prenez le coéfficient de perte à l'entrée et la sortie du paille come être $K_e = K_s = 0.5$. Négliger la viscosité dans le vapeur. Quelle doit-être le rapport entre le diamètre maximal et minimal du Venturi pour supporter un flux de lait de 12 ml/s?



Question 4:

Pour ce problème, utilisez les données que vous avez collectées dans le laboratoire de pipe-flow.

a) Pour le plus petit tuyau, calculer le coefficient de frottement à chaque débit. Utilisez le diagramme de Moody pour estimer la rugosité relative du tuyau.

b) Répéter la question 1 pour l'autre diamètre du tuyau utilisé.

c) Répétez la question 1 pour le tuyau rugueux.

d) Si certains des coefficients de frottement calculés s'écarte du valeur attendue, pourquoi? Est-ce que toutes les hypothèses utilisées dans leur dérivation sont valides?

e) Calculer le coefficient de perte pour les deux coudes aux trois débits.

f) Utiliser les différences de pression dans les débitmètres Venturi et Pitot pour calculer le débit. Est-il d'accord avec la valeur mesurée à l'aide du débitmètre numérique?

For this probelem, use the data you collected in the pipe-flow lab.

1. For the smallest pipe, compute the friction factor for this situation at each flow rate. Use the Moody diagram to estimate the relative roughness of the pipe.
2. Repeat question 1 for the other diameter of pipe used.
3. Repeat question 1 for the rough pipe.
4. If any of the computed friction factors deviate from the expected value, why might that be? Are all the assumptions used in their derivation valid?
5. Compute the loss coefficient for the two elbows at all three flow rates.
6. Use the pressure differences in the Venturi and Pitot flow meters to compute the flow rate. Does it agree with the value measured using the digital flow meter?

Table 1: Typical roughness of common engineering materials

Material	Roughness, ϵ (mm)
Riveted steel	0.9–9
Concrete	0.3–3
Wood stave	0.2–0.9
Cast iron	0.26
Galvanized iron	0.15
Asphalted cast iron	0.12
Commercial steel or wrought iron	0.046
Drawn tubing	0.0015

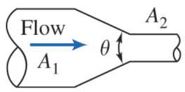
Representative Dimensionless Equivalent Lengths (L_e/D) for Valves and Fittings

Fitting Type	Equivalent Length,[a] L_e/D
Valves (fully open)	
Gate valve	8
Globe valve	340
Angle valve	150
Ball valve	3
Lift check valve: globe lift	600
angle lift	55
Foot valve with strainer: poppet disk	420
hinged disk	75
Standard elbow: 90°	30
45°	16
Return bend, close pattern	50
Standard tee: flow through run	20
flow through branch	60

[a]Based on $h_{l_m} = f(L_e/D)(\bar{V}^2/2)$.

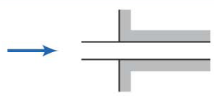
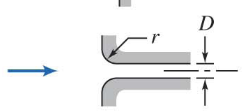
Figure 1: Equivalent lengths for certain geometries; Pritchard (2011).

Loss Coefficients (K) for Gradual Contractions: Round and Rectangular Ducts

		Included Angle, θ, Degrees						
	A_2/A_1	10	15–40	50–60	90	120	150	180
	0.50	0.05	0.05	0.06	0.12	0.18	0.24	0.26
	0.25	0.05	0.04	0.07	0.17	0.27	0.35	0.41
	0.10	0.05	0.05	0.08	0.19	0.29	0.37	0.43

Note: Coefficients are based on $h_{l_m} = K(\vec{V}_2^2/2)$.

Minor Loss Coefficients for Pipe Entrances

Entrance Type		Minor Loss Coefficient, K[a]
Reentrant		0.78
Square-edged		0.5
Rounded		r/D: 0.02, 0.06, ≥ 0.15; K: 0.28, 0.15, 0.04

[a]Based on $h_{l_m} = K(\overline{V}^2/2)$, where $\overline{V}$ is the mean velocity in the pipe.

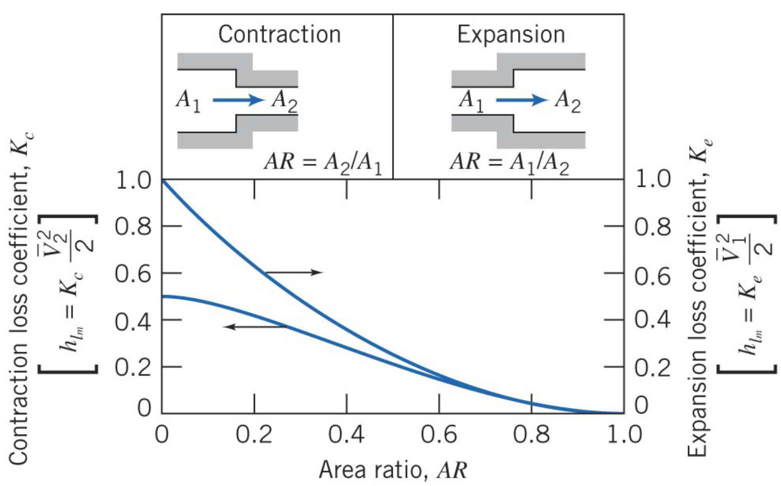


Figure 2: Loss coefficients for certain geometries; Pritchard (2011).

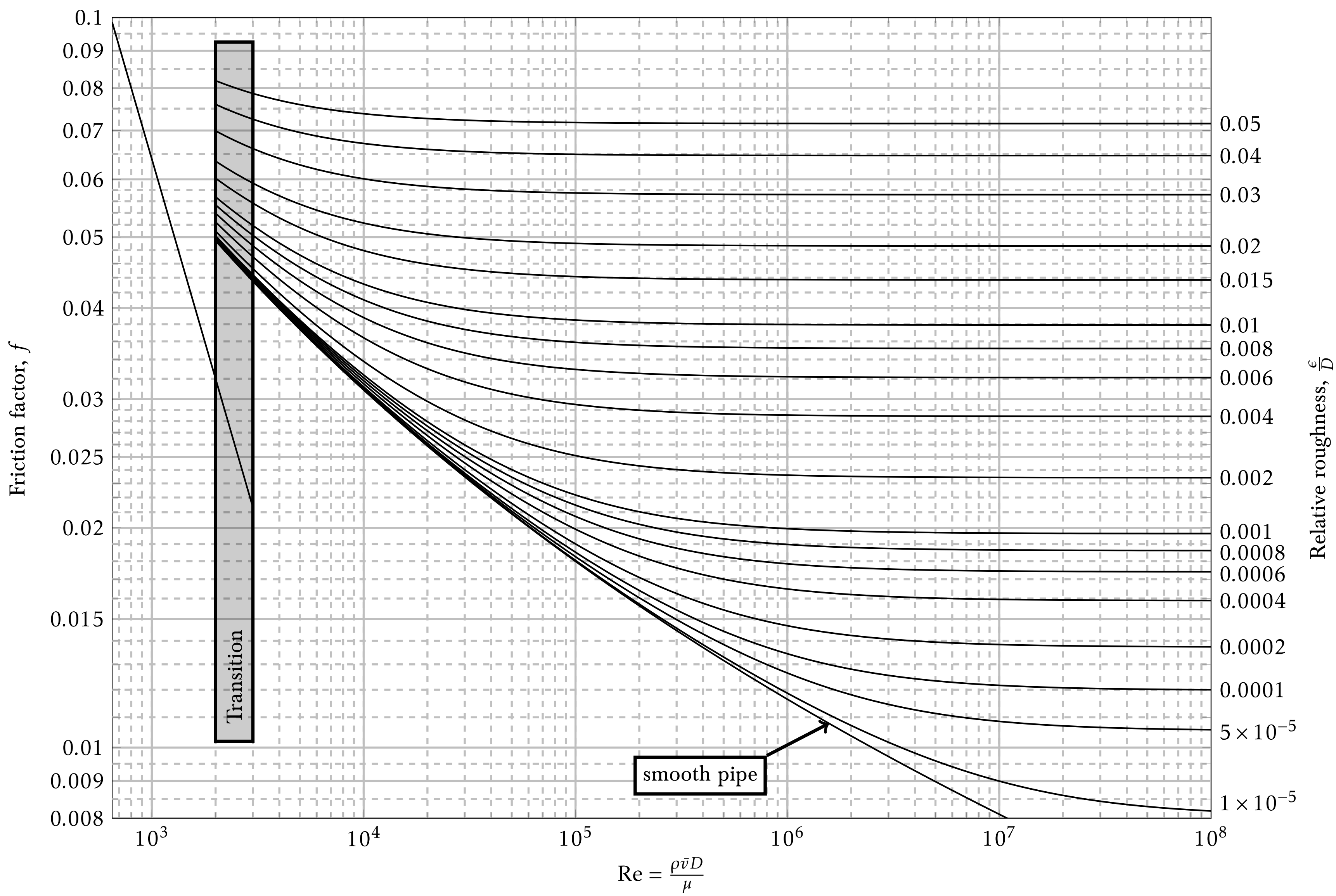


Figure 3: Moody diagram

Name:

5) A piping system is used to deliver water from a lake to a storage tank, as shown. The piping system is made of commercial steel and is 20 m long in total. The pipes have an inner diameter of 4 cm. The system has an electric pump and three 90° elbows. The pressure difference provided by the pump is 66 kPa. The viscosity of the water is, $\mu_{H_2O} = 1.3\times10^{-3}$ Pa s. Treat the pipe entrace as "reentrant" (see page 11).

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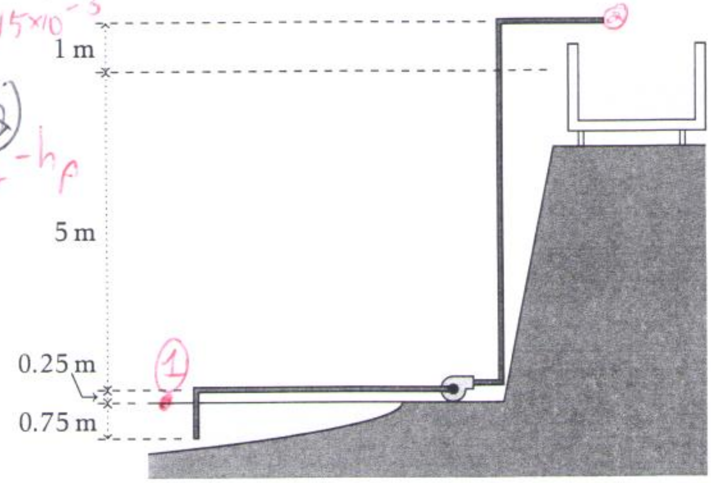
b) If the pump has an efficiency of 75%, how much power does it consume?



$\varepsilon = 0.046\text{ mm}$ $\quad \frac{\varepsilon}{D} = \frac{0.046\text{ mm}}{40\text{ mm}} = 1.15\times10^{-3}$

$$\left(\frac{P}{\rho}+\alpha\frac{\bar V^2}{2}+gz\right)_1 - \left(\frac{P}{\rho}+\alpha\frac{\bar V^2}{2}+gz\right)_2 = h_{lt}-h_p \quad (3)$$

$$h_p = \frac{\Delta p}{\rho} = \frac{66000\text{ Pa}}{1000\text{ kg/m}^3} = 66\text{ m}^2/\text{s}^2 \quad (2)$$

$$h_{lt} = f\frac{L}{D}\frac{\bar V^2}{2} + 3f\frac{L_e}{D}\frac{\bar V^2}{2} + K\frac{\bar V^2}{2} \quad (2)$$

$$-\left(\frac{\alpha\bar V^2}{2}+gz_2\right) = \frac{\bar V^2}{2}\left[f\left(\frac{L}{D}+3\frac{L_e}{D}\right)+K\right] - 66\text{ m}^2/\text{s}^2$$

$$\underbrace{-gz_2 + 66\text{ m}^2/\text{s}^2}_{(9.81\text{ m/s}^2)(6.25\text{ m})} = \frac{\bar V^2}{2}\left[f\left(\underbrace{\frac{L}{D}}_{\frac{20\text{ m}}{0.04\text{ m}}=500}+\overbrace{3\frac{L_e}{D}}^{30}\right)+\overbrace{K}^{0.78}+\alpha\right]$$

$$4.6875\text{ m}^2/\text{s}^2 = \frac{\bar V^2}{2}\left[f(590)+\alpha+0.78\right]$$

$$\bar V = \sqrt{\frac{2(4.6875\text{ m}^2/\text{s}^2)}{f(590)+\alpha+0.78}}$$

assume $Re = 100\,000$
$f = 0.022$
$\alpha = 1$

4.6875

$$\bar V = \sqrt{\frac{2(\text{ m}^2/\text{s}^2)}{0.022(590)+1.78}} = 0.8\text{ m/s} \rightarrow Re = 24600$$

$f = 0.027$
$\alpha = 1$

$$\bar V = \sqrt{\frac{2(4.6875\text{ m}^2/\text{s}^2)}{0.027(590)+1.78}} = 0.73\text{ m/s} \rightarrow Re = 22500$$

close enough

$\bar V = 0.73\text{ m/s}$

$$Q = \bar V A = \frac{\bar V \pi D^2}{4} = (0.73\text{ m/s})\pi\frac{(0.04\text{ m})^2}{4}$$

$$\boxed{= 9.2\times10^{-4}\text{ m}^3/\text{s} = 0.92\text{ L/s}} \quad (2)$$

$$h_p = \frac{\dot W}{Q\rho}$$

$$\dot W = h_p Q\rho = 66\text{ m}^2/\text{s}^2(9.2\times10^{-4})(1000) = 60.72\text{ W} \quad (2)$$

$$\dot W = \frac{60.72}{.75}$$

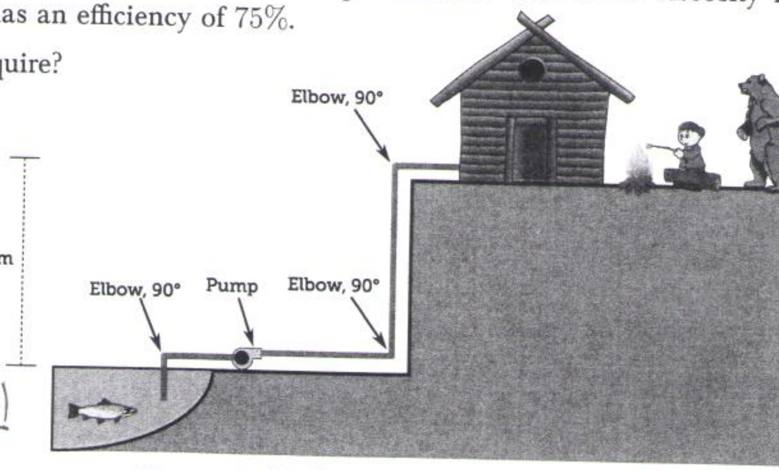
$$\boxed{= 81\text{ W}}$$

Name:

5) A piping system is used to deliver water from a lake to a cabin, which is situated 5 meters higher than the lake level. A 30-m long piping system made of commercial steel is used, with an internal diameter of 2.5 cm. The system has three 90° elbows. The delivery water pressure required at the cabin entrance is 300 kPa (gauge) and the required volume flow rate is 20 liters per minute. The water viscosity is $\mu_{H_2O} = 1.3 \times 10^{-3}$ Ns/m^2. The pump has an efficiency of 75%.

What electrical power does the pump require?

Neglect minor losses at the pipe entrance.



$$\bar{V} = \frac{Q}{S} = \frac{20\,L/min}{\frac{\pi}{4}(0.025m)^2} = \frac{3.33\times10^{-4}\,m^3/s}{4.91\times10^{-4}\,m^2}$$

$$\boxed{= 0.68\,m/s}\quad 1$$

$$Re = \frac{\rho\bar{V}D}{\mu} = \frac{(1000\,kg/m^3)(0.68\,m/s)(0.025\,m)}{1.3\times10^{-3}\,Ns/m^2}$$

$$1\quad \boxed{= 13077} \quad \therefore \text{turbulent}$$

$$\frac{\varepsilon}{D} = \frac{0.046\,mm}{25\,mm} = 0.00184 \quad 1$$

$$\boxed{f = 0.032}\quad 1$$

$$h_{\ell t} = f\frac{L}{D}\frac{\bar{V}^2}{2} + 3f\left(\frac{L}{D}\right)_E\frac{\bar{V}^2}{2} = f\frac{\bar{V}^2}{2}\left[\frac{L}{D} + 3\underset{30}{\left(\frac{L}{D}\right)_E}\right]$$

$$= (0.032)\frac{(0.68\,m/s)^2}{2}\left[\frac{30m}{0.025m} + 3(30)\right] = \boxed{9.54\,m^2/s^2}\quad 1$$

$$\left(\cancel{\frac{P_A}{\rho}}^{\,0} + \alpha\cancel{\frac{\bar{V}_A^2}{2}}^{\,0} + \cancel{gz_A}^{\,0}\right) - \left(\frac{P_B}{\rho} + \alpha\frac{\bar{V}_B^2}{2} + gz_B\right) = h_{\ell t} - h_p \quad 1$$

$$h_p = h_{\ell t} + \frac{P_B}{\rho} + \alpha\frac{\bar{V}_B^2}{2} + gz_B = 9.54\,m^2/s^2 + \frac{300\,000\,Pa}{1000\,kg/m^3} + \frac{(0.68\,m/s)^2}{2} + 9.81\,m/s^2(5m)$$

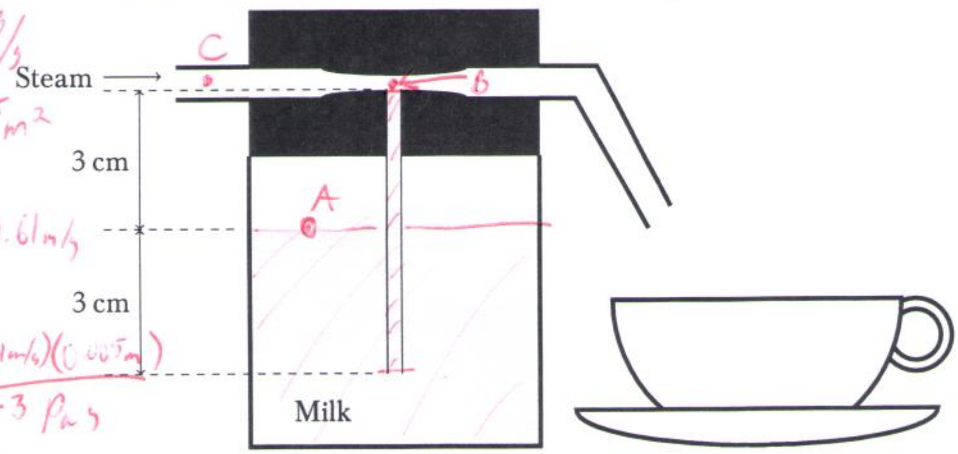
$$2\quad \boxed{= 358.82\,m^2/s^2}$$

$$\dot{W}_p = h_p\dot{m} = (358.82\,m^2/s^2)(1000\,kg/m^3)(3.333\times10^{-4}\,m^3/s) = \boxed{120W}\quad 1$$

$$\dot{W}_{provided} = \frac{\dot{W}_p}{\eta} = \frac{120W}{0.75} = \boxed{160W}\quad 1$$

Name:

2) An automatic cappuccino machine uses a Venturi to draw milk, $\rho_{\text{milk}} = 1035\,\text{kg/m}^3$, vertically into a stream of hot steam through a straw with a diameter of 0.5 cm and a length of 6 cm. The viscosity of the milk is $\mu_{\text{milk}} = 3 \times 10^{-3}\,\text{Pa s}$. The steam has a density, $\rho_{\text{steam}} = 1.7\,\text{kg/m}^3$ and it enters the Venturi with a relative pressure of 200 Pa and velocity of 3 m/s. Assume the loss coefficient, K, at both the straw entrance and exit is 0.5. Neglect viscosity in the steam flow. What must be the ratio between the maximum and minimum diameters in the Venturi to achieve a milk flow of 12 ml/s?



$Q = 12\,\text{ml/s} = 1.2\times10^{-5}\,\text{m}^3/\text{s}$

$A = \frac{\pi}{4}(0.005\,\text{m})^2 = 1.96\times10^{-5}\,\text{m}^2$

$V = \frac{Q}{A} = \frac{1.2\times10^{-5}\,\text{m}^3/\text{s}}{1.96\times10^{-5}\,\text{m}^2} = 0.61\,\text{m/s}$

$Re = \frac{\rho V D}{\mu} = \frac{(1035\,\text{kg/m}^3)(0.61\,\text{m/s})(0.005\,\text{m})}{3\times10^{-3}\,\text{Pa s}}$

$= 1052 \rightarrow$ laminar

$f = \frac{64}{Re} = \frac{64}{1052} = 0.06$ (2)

$\left(\cancel{\frac{P_A}{\rho}}^{0} + \cancel{\alpha\frac{V_A^2}{2}}^{0} + \cancel{g z_A}^{0}\right) - \left(\frac{P_B}{\rho} + \alpha\frac{V_B^2}{2} + g z_B\right) = \frac{64}{Re}\frac{L}{D}\frac{V_B^2}{2} + 2K\frac{V_B^2}{2}$

$P_B = -\rho\left\{\left[\frac{64}{Re}\frac{L}{D} + 2K + \alpha\right]\frac{V_B^2}{2} + g z_B\right\}$

$\alpha = 2$

$= -(1035\,\text{kg/m}^3)\left\{\left[\frac{64}{1052}\frac{(0.06\,\text{m})}{(0.005\,\text{m})} + 2(0.5) + 2\right]\frac{(0.61\,\text{m/s})^2}{2} + 9.81\,\text{m/s}^2 \times 0.03\,\text{m}\right\}$

$P_B = -1023\,\text{Pa}$ (3)

$\frac{P_C}{\rho_{\text{steam}}} + \frac{V_C^2}{2} = \frac{P_B}{\rho_{\text{steam}}} + \frac{V_B^2}{2}$

$V_B = \sqrt{2\left(\frac{P_C - P_B}{\rho_{\text{steam}}} + \frac{V_C^2}{2}\right)}$

$V_B = \sqrt{2\left(\frac{200\,\text{Pa} - (-1023\,\text{Pa})}{1.7\,\text{kg/m}^3}\right) + \frac{(3\,\text{m/s})^2}{2}}$

$V_B = 38\,\text{m/s}$ (3)

$\frac{A_C}{A_B} = \frac{V_B}{V_C} = \frac{38\,\text{m/s}}{3\,\text{m/s}} = 12.7$

$\left(\frac{D_C}{D_B}\right) = \sqrt{\frac{A_C}{A_B}} = \sqrt{12.7} = \boxed{3.6}$ (2)

4) a) b) c) $\frac{\Delta p}{\frac{1}{2}\rho\bar{V}^2} = f \frac{L}{D}$

$$f = \frac{\Delta p}{\frac{1}{2}\rho\bar{V}^2}\frac{D}{L} \qquad Re = \frac{\rho\bar{V}L}{\mu}$$

look up ρ & μ for water at measured temperature.

d) Only the smallest pipe had an entry region long enough for fully-developed flow.

e) $$\frac{\Delta p}{\rho} = K\frac{\bar{V}^2}{2} \qquad K = \frac{\Delta p}{\frac{1}{2}\rho\bar{V}^2}$$

f) pitot: $p_1 + \frac{1}{2}\rho V_1^2 = p_2$

$$p_2 - p_1 = \Delta p = \frac{1}{2}\rho V^2$$

$$V = \sqrt{\frac{2\Delta p}{\rho}} \qquad Q = VS = \frac{\pi D^2}{4}\sqrt{\frac{2\Delta p}{\rho}}$$

Venturi:

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$Q = v_1 S_1 = v_2 S_2$$

$$p_1 + \frac{1}{2}\rho\left(\frac{Q}{S_1}\right)^2 = p_2 + \frac{1}{2}\rho\left(\frac{Q}{S_2}\right)^2$$

$$\Delta p = \frac{1}{2}\rho Q^2\left[\left(\frac{1}{S_2}\right)^2 - \left(\frac{1}{S_1}\right)^2\right]$$

$$Q = \sqrt{\frac{2\,\Delta p}{\rho\left[\left(\frac{1}{S_2}\right)^2 - \left(\frac{1}{S_1}\right)^2\right]}}$$

$$= S_1\sqrt{\frac{2\,\Delta p}{\rho\left[\left(\frac{S_1}{S_2}\right)^2 - 1\right]}}$$

$$= \frac{\pi D_1^2}{4}\sqrt{\frac{2\,\Delta p}{\rho\left[\left(\frac{D_1}{D_2}\right)^4 - 1\right]}}$$